

# Transmission Power Adaptation Based Energy Efficient Neighborhood Discovery

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**Abstract**—Neighborhood discovery - detection of the devices within communication range using HELLO messages - is a fundamental mechanism in wireless sensor networks (WSN) which enables usage of many different topology control and routing algorithms. Even though it is very important, most of the algorithms does not take into account parameters of the neighborhood discovery. We present two algorithms that adapt power of transmission of the sensors in a mobile WSN by still adapting frequency of HELLO messages in order to save energy and get accurate neighborhood tables. First solution is based on the knowledge of turnover - change in the number of neighbors in consecutive iterations of the neighborhood discovery - used in conjunction with adaptation of frequency and transmission range, minimizing general cost of transmission of the HELLO messages. Second solution is based on computing of optimal range. Both algorithms are based on theoretical analysis. Results show that they are energy efficient and outperform solutions of the literature.

**Index Terms**—wireless sensor network, neighborhood discovery, HELLO message, transmission power adaptation, turnover;

## I. INTRODUCTION

Wireless sensor networks (WSN) can be defined as networks of small spatially distributed devices, called sensor nodes, which are working cooperatively - exchanging messages wirelessly - on the same application. Mobility of the sensor nodes is emerging problem and nowadays it becomes more and more under the attention of scientific community. Included in the WSN, mobility arises new problems and questions such as optimization of energy consumption, connectivity of the WSN, routing in mobile networks and many more. Nodes in WSN can have mobility included inherently, as it is case with nodes which are attached to the animals with the purpose to track animals' habit and their natural habitat<sup>1</sup>. In this case mobility pattern is very often hard to estimate and protocols built upon this problem must take into account possible losses of connectivity or delays in the transmission of messages. Other possibility would include mobile agents (robots) which may have given mobile pattern to follow or even use it to improve some of the parameters in the WSN [1]. In this case overall energy efficiency of transmission can be significantly augmented using controlled mobility smart placement of nodes related to the routing path. Due to the specific nature mobile networks some of the characteristic mechanisms used in static WSNs need to be redefined and adapted to the specific types of the mobility of the nodes.

Neighborhood discovery is one of the most important protocols in functioning of WSN. Mechanism behind this protocol is rather simple, it includes periodical sending of specific type of the messages, called *HELLO messages* (also known as *beacon messages*) and gathering the data from the received HELLO messages. Hello messages contain the data of the sender id, unique identification number for the node in the WSN - usually MAC address in practical applications. Each node, usually, acquires data from all HELLO messages that it has received and organizes them into the *neighborhood table* which can be further used for some kind of topology control [2] or proactive routing [3]. The main challenge, in the practical usage of the neighborhood discovery mechanism is the accuracy of the neighborhood table, size of HELLO messages, as well as their content and the frequency at which they are being sent in order not to spent network resource uselessly. Indeed if the Hello frequency is too low, nodes may not be detected by their neighbors, leading deprecated neighborhood tables, and protocol failures are likely to occur. But if the frequency is too high, neighborhood tables are up to date, but then energy and bandwidth are wasted to the detriment of data traffic. Similarly, nodes can adapt their range. The longer range, the more neighbors to be detected by the more energy is spent. It is shown [4] that varying of these parameters with other parameters of the protocol stack in WSN (MAC layer, clustering) can have significant impact on the performance of the nodes and whole network. In the presence of the mobility of the nodes we must observe that each of the nodes in successive periods of time  $\Delta t$  is going to receive HELLO messages of the different nodes and also HELLO messages that the node itself is sending are going to be received by different sets of nodes. This means that neighborhood table is changing due to the mobility of the nodes. Question that arises is: how can we use mechanism of neighborhood discovery in the presence of the mobility and which parameters are the best for usage in these cases?

Our work, presented in this paper, relies on the Turnover based Adaptive Hello Protocol [5] which exploits the idea of existence of optimal frequency of hello messages and using this result and the change of the neighborhood table, called *turnover* gives the optimal value for the turnover in the mobile WSNs and sets the basis for the future works which can include prediction of the mobility or some kind of topology control in the mobile WSNs. Coming from the fact that overall energy efficiency can be minimized when manipulating both power of transmission and the frequency of hello

<sup>1</sup><http://wasp-project.org>

messages, we present two algorithms for the adaptation of the frequency of hello messages in parallel with the adaptation of the power of transmission of each node. Both algorithms rely on theoretical analysis. They periodically executed by each node (independently of the other nodes) in the mobile network, upon gathering the data from the hello messages when node receives them. To the traditional neighborhood table we have added also history table in which we are keeping successive neighborhood tables with the idea to track the changes between the neighborhood tables and according to those changes properly changes the factors of neighborhood discovery and transmission range of each node. Results show that our solutions maintain same accuracy as TAP protocol but saving energy.

Section II gives review of the literature related to our work. Section III presents our algorithms while Section V provides theoretical preliminaries and analysis. Simulation results are detailed in Section VI. Section VII gives conclusion and directions of future work.

## II. RELATED WORK

Hello protocol is first introduced in OSPF [6] and works as follows. Each node is sending HELLO messages at fixed frequency, noted  $f_{HELLO}$  to allow other nodes knowledge of its presence, and at the same time each node collects data from HELLO messages that it receives from other nodes *i.e.* if node  $u$  for example receives HELLO message from the node  $v$  it puts node  $v$  in his neighborhood table, similarly if node  $v$  receives HELLO message from node  $u$  he will put it in his neighborhood table.

Neighborhood discovery mechanism as described in here assumes that  $f_{HELLO}$  is fixed. In mobile networks this assumption is not adequate since it is more natural to assume that the nodes which are moving faster are also changing their neighbors faster thus they need to update neighborhood table more often but also since they are losing (and gaining) their neighbors more often it is logical that they send HELLO messages at a higher rate. In this case when we talk about mobility, we think of the relative mobility of the node itself referring to the other nodes. For example if we have a fleet of the nodes which are moving together at some fixed speed then they are in the relative sense static because each of the node always "sees" the same nodes around him. In [7], assuming that there is a knowledge of relative speed between two nodes  $S$  and threshold distance in communication area  $aR$  such that  $a < 1$ , is given the optimal value for the frequency of HELLO messages:

$$f_{opt} = \frac{2S}{aR} \quad (1)$$

There exist several works which were trying to adapt frequency of HELLO messages using the information of position and speed thus assuming the presence of these devices on the sensor nodes. Turnover based adaptive hello protocol (TAP) [5] is the first algorithm which adapts frequency of HELLO messages without using any additional hardware. In this paper authors are proposing usage of *turnover* value which can be

calculated considering the differences of two neighborhood tables at fixed moments in time. The first neighborhood table is the current one and the other is the one obtained and saved at the previous moment. This protocol thus only requires periodical save of current neighborhood table in so called *history* table which will be then used for the calculation of the turnover.

In this case we come to the question how often we should update neighborhood table and which of the values should be considered as obsolete. Neighborhood lifetime algorithm (NLA) [8] works alongside the TAP and it adapts refreshing the entries of the neighborhood table using the information of the speed of the nodes and frequency of HELLO messages.

Knowledge of the relative speed of the nodes can also lead into the detection and estimation of the type of the mobility. In [9] authors are presenting Autoregressive Hello protocol (ARH) and evaluate its performance alongside TAP. Different approach is presented in [10] where the authors are relying on the signal strength descriptors which are embedded in wireless radio (again without using any additional hardware) to detect mobility of the nodes in the closed space (room). While giving excellent results in detection and prediction of mobility these works do not tackle the possibility of changing the transmission range of the nodes.

Distinction is made between initial and continuous neighborhood discovery [11]. Initial neighborhood discovery is applied when the sensor is unaware of its immediate environment, while the continuous neighborhood discovery is performed when the node is already aware of the neighborhood. Initial neighborhood discovery needs to be done by each sensor separately while continuous discovery can be applied as joint task of the segment of the neighborhood, and not each sensor. This joint task allows single node to spend more time in sleep mode and relies to the part of its neighborhood which is currently in active state. In this way they can lower energy consumption with high probability that new node is going to be properly discovered. The algorithm guarantees that new node is going to be discovered in given time slot with requested probability,

The best of our knowledge the algorithm that we are presenting is the first that tries to handle both frequency of the HELLO messages and the transmission range of the nodes when nodes are not aware of their position.

## III. ALGORITHMS FOR ADAPTATION OF FREQUENCY AND TRANSMISSION RANGE

In this section we present our algorithms used to adapt both frequency of HELLO messages and node transmission range. The first algorithm is based on [5] which relies on the calculation of turnover. The turnover of [5] has been redefined to capture both unilateral and bilateral neighbors (that appear because of different transmission ranges of each node) along with optimal frequency of HELLO messages [7]. This turnover helps in determining the HELLO frequency. Algorithm is then declined in two variants to define the transmission range. Second algorithm is combining results for optimal frequency

and dependency between  $f$  and transmission range to adapt both frequency and transmission range without knowledge of turnover.

Both algorithms are based on the existence of an optimal frequency of HELLO messages [7] given by Eq. ??.

#### A. Using the turnover

To be able to define this algorithm first step for us is to calculate turnover,  $r$ , in a real scenario. Number of new neighbors (theoretically calculated in previous section) is translated into the calculation of number the new neighbors with given weights.

- unilateral neighbors – the situation when the node finds new unilateral neighbor; it is quantified with  $\theta_{uni}$  multiplied by the total number of new unilateral neighbors in given period of time,
- bilateral neighbors – the situation in which the node finds new bilateral neighbor; factor  $\theta_{bi}$  is used to quantify new bilateral neighbor which is found and it is multiplied by total number of new bilateral nodes  $y$  found in given period of time.

Current turnover,  $r$ , is calculated by each node independently according to their own track of the changes between the types of the links, using equation:

$$r = \frac{\theta_{uni} \cdot x + \theta_{bi} \cdot y}{x_t + y_t} \cdot \frac{T_{HELLO}}{\Delta t} \quad (2)$$

where  $\theta_{uni} = 1$ ,  $\theta_{bi} = 2$ ,  $\Delta t$  is the time passed between updates of two tables that we are comparing,  $T_{HELLO}$  is the period of HELLO messages,  $x_t + y_t$  presents total number of neighbors in neighborhood table and it is the sum of all unilateral and bilateral neighbors. These specific values for the  $\theta$  parameters are used with the respect of the type of neighbor. For new unilateral neighbors we use smaller value of  $\theta$  because we consider those links weaker and we want to give them less importance in calculation of turnover. Bilateral neighbors are considered stronger and they are multiplied by bigger  $\theta$ .

First solution is detailed in Algorithm 1 *Turnover based Power Transmission Adjustment*. Algorithm 1 is executed by each node independently only based in the observation of its neighborhood. Algorithm 1 is TAP-fashion algorithm which adapt the hello frequency dynamically based on changes on node neighborhood. It aims to reach an optimal turnover previously computed thanks to computing of new neighbors provided in Section ?. Starting point for the algorithms are neighborhood table and *history table*, and all other values are calculated using these values, including the turnover. Neighborhood table is the standard neighborhood table, as explained in Section ?. History table is table in which we are preserving the values which we have obtained for a given time moment, number of new bilateral and unilateral neighbors and the changes between the type of the neighbors which allows us to calculate the turnover at that moment.

Adjustment of  $f$  is calculated through the period between two HELLO messages  $d_{HELLO}$ , where  $f = \frac{1}{d_{HELLO}}$ :

$$d_{HELLO} = \begin{cases} d_{HELLO} + \frac{d_{HELLO}}{4} \cdot g(r) & \text{if } r \leq r_{opt} \\ d_{HELLO} - \frac{d_{HELLO}}{4} \cdot g(r) & \text{otherwise} \end{cases} \quad (3)$$

Function  $g(r)$  is retrieved using turnover:

$$g(r) = \begin{cases} \left(\frac{r-r_{opt}}{r_{opt}}\right)^2 & \text{if } r < 2 \cdot r_{opt}, \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

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#### Algorithm 1 Turnover based Power Transmission Adjustment

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1: while 1 do
2:   CalculateCurrentTurnover  $r = \frac{\theta_{uni} \cdot x + \theta_{bi} \cdot y}{x_t + y_t} \cdot \frac{T_{HELLO}}{\Delta t}$ 
3:   if  $r \leq r_{opt}$  then
4:     Lower  $f$  with Eq. 3
5:   else if  $r > r_{opt}$  then
6:     Augment  $f$  with Eq. 3
7:   end if
8:   Adapt Power of Transmission
9: end while

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Both variants differ in the call to *Adapt Power of Transmission* function in Line 8 of Algorithm 1. First variant uses Eq. 1 while second variant uses the analysis presented in previous section. This analysis as a result gives us  $R = \sqrt[\alpha]{\frac{C_1}{f}} - C$  as function for transmission range.

#### IV. MINIMIZING THE ENERGY CONSUMPTION

This second algorithm is based on the idea that there exists a joint way to optimize the transmission range and the Hello frequency. Indeed, the energy spent in period  $\Delta t$  of time by a node  $u$  can be expressed as the number of messages sent by  $u$  in  $\Delta t$  multiplied by the cost of a message. The number of messages sent by  $u$  during  $\Delta t$  is  $\Delta t \cdot f_u(R_u, t)$  where  $f_u(R_u, t)$  is the HELLO frequency of node  $u$  and  $R_u$  is the range of node  $u$  at time  $t$ . We assume that the cost of a message follows energetic model given in [] with  $E(R_u) = R_u(t)^\alpha + C$ , where  $\alpha$  is a real constant ( $\geq 1$ ) that represents the signal attenuation and  $c$  is the overhead due to signal processing. We assume that  $\Delta t$  is such that  $u$  does not change its range nor its frequency during this period of time, so  $R_u(t) = R$ . Thus, energy spent by node  $u$  with transmission range  $R$  during  $\Delta t$  is:

$$cost_{\Delta t}(R) = \Delta t \cdot f(R) \times (R^\alpha + C) \quad (5)$$

Based on this equation and combining it to Eq. 1, we obtain a system with two equations and two unknowns:  $R$  and  $f$  that we compute in Section V-C.

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#### Algorithm 2 Cost Based Transmission Power Adjustment

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1: To get  $R$ , solve  $R^\alpha - \frac{\alpha RC_1}{2S} + C = 0$  based on  $\alpha$  values.
2:  $f = \frac{2S}{\alpha R}$ 
3: Return ( $f, R$ )

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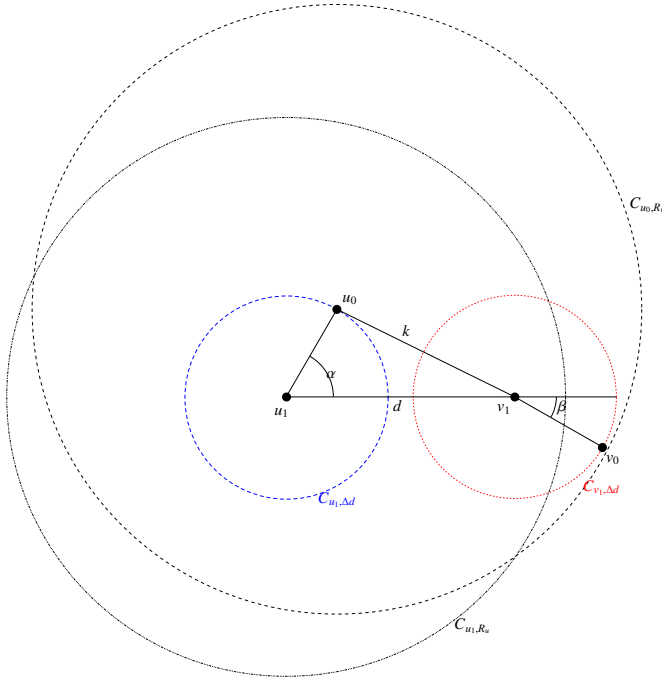


Fig. 1. Global view. Circle  $C_{u,R_u}$  is centered at the position of the node  $u$  with radius  $R_u$ . In this case, node  $v$ , with radius  $R_v$  is a new neighbor if and only if  $v_0$  does not lie in the area delimited by  $C_{u_0,R_u}$  and if  $v_1$  lies in the area delimited by  $C_{u_1,R_u}$ . The blue dashed circle  $C_{u_1,\Delta d}$  and the red dotted circle  $C_{v_1,\Delta d}$  represent the possible positions of the  $u_0$  and  $v_0$ .

## V. THEORETICAL ANALYSIS

### A. Model and notations

Wireless networks are presented by a graph  $G = (V, E)$  in which  $V$  is set of nodes and  $E$  is set of edges such that  $E \subseteq V^2$ . We suppose that nodes are randomly deployed using a Poisson Point Process [12], with node positions which are independent and  $\lambda > 0$ , where  $\lambda$  represents the mean number of nodes per surface unit. Each node  $u$  has transmission range  $R_u$  such that  $0 < R_u < R_{max}$ .

In this paper, since nodes can adapt their range, they do not have necessarily the same range. So we differentiate two types of neighbors:

- *bilateral neighbors* – two nodes  $u$  and  $v$  with property  $|uv| < \min(R_u, R_v)$
- *unilateral neighbors* – node  $u$  is unilateral neighbor of node  $v$  if  $\min(R_u, R_v) < |uv| < \max(R_u, R_v)$

Every node is moving at constant speed  $S$  in random direction. Position of node  $u$  at the moment  $t_0$  is given as  $u_0$  (respectively node  $v$  at  $t_0$  has position  $v_0$ ) and at the moment  $t_1$  position is  $u_1$  (respectively  $v_1$ ). Distance covered by a node during the time  $\Delta t$  is given as  $\Delta d = V \times \Delta t$ . Angles  $\alpha$  and  $\beta$  are given as  $\angle \overrightarrow{u_1 v_1}, \overrightarrow{u_1}, \overrightarrow{u_0}$  and  $\angle \overrightarrow{u_1 v_1}, \overrightarrow{v_1}, \overrightarrow{v_0}$  respectively, and they represent the directions from which nodes  $u$  and  $v$  come. In the worst case, node direction is random and thus  $\alpha$  and  $\beta$  are uniformly distributed in  $[-\pi, \pi]$ .

### B. Theoretical analysis on probable number of new neighbors

In our analysis, we are interested in the mean number of new neighbors that node  $u$  is going to meet during the time interval  $\Delta t$ . We focus on the typical node  $u$ . Let  $N_{bi}(u)_{\Delta t}$  be the number of the bilateral neighbors of node  $u$  and  $N_{uni}(u)_{\Delta t}$  the number of unilateral neighbors of node  $u$  that node  $u$  detects during period  $\Delta t$ . Let  $v$  be a node at distance  $d$  ( $d < R_{max}$ ) from node  $u$  at time  $t_1 = t_0 + \Delta t$ . We note:

- $P^{bi}$ : the probability that node  $v$  is new bilateral neighbor of node  $u$
- $P^{uni}$ : the probability that node  $v$  is new unilateral neighbor of node  $u$

From this we can determine the average values of  $N_{bi}(u)_{\Delta t}$  and  $N_{uni}(u)_{\Delta t}$ :

$$E[N_{bi}] = \int_{R_u=0}^{R_{max}} \int_{R_v=0}^{R_{max}} \int_{d=0}^{R_u} P(R_u)P(R_v)\lambda\pi d \times P^{bi}(d, R_u, R_v) d d d R_u d R_v \quad (6)$$

$$E[N_{uni}] = \int_{R_u=0}^{R_{max}} \int_{R_v=0}^{R_{max}} \int_{d=0}^{R_u} P(R_u)P(R_v)\lambda\pi d \times P^{uni}(d, R_u, R_v) d d d R_u d R_v \quad (7)$$

where  $P(R_u)$  is probability that  $u$  has radius  $R_u$  and  $P(R_v)$  is probability that  $v$  has radius  $R_v$ . Figure 1 illustrates our model in case when  $R_u < R_v$ . Circle  $C_{u,R_u}$  is centered at node  $u$  with radius  $R_u$ . In this case, node  $v$ , with radius  $R_v$  is a new neighbor if and only if  $v_0$  does not lie in the area delimited by  $C_{u_0,R_u}$  and if  $v_1$  lies in the area delimited by  $C_{u_1,R_u}$ . The blue dashed circle  $C_{u_1,\Delta d}$  and the red dotted circle  $C_{v_1,\Delta d}$  represent the possible positions of  $u_0$  and  $v_0$ .

1) *Computing the number of new bilateral neighbors*  $E[N_{bi}]$ : We are interested in the probability that at time  $t_0$ ,  $u$  and  $v$  were either not neighbors ( $|u_0 v_0| > \max(R_u, R_v)$ ) or only unilateral neighbors ( $\max(R_u, R_v) > |u_0 v_0| > \min(R_u, R_v)$ ). We are interested in probability  $P^{bi}$  that given  $R_u$ ,  $R_v$  and  $d$ ,  $|u_0 v_0| > \min(R_u, R_v)$  knowing that  $|u_1 v_1| \leq \min(R_u, R_v)$ .

We make the distinction between two cases:

- **Case 1:  $R_u \leq R_v$**  We note  $P_{R_u \leq R_v}^{bi}$  the probability that  $v$  is a new bilateral neighbor of  $u$  if  $R_u \leq R_v$ ; for this case  $P(R_v) = \frac{1}{R_{max}}$  and  $P(R_u | R_u < R_v) = \frac{R_v}{R_{max}}$ ,
- **Case 2:  $R_u > R_v$**  We note  $P_{R_u > R_v}^{bi}$  the probability that  $v$  is a new bilateral neighbor of  $u$  if  $R_u > R_v$ ; for this case  $P(R_v) = \frac{1}{R_{max}}$  and  $P(R_u | R_u \leq R_v) = \frac{R_{max} - R_v}{R_{max}}$ .

From these two cases we have:

$$\begin{aligned} E[N_{bi}] &= \int_{R_u=0}^{R_{max}} \int_{R_v=0}^{R_{max}} \int_{d=0}^{R_u} P(R_u)P(R_v)\lambda\pi d \times P^{bi}(d, R_u, R_v) d d d R_u d R_v \\ &= \int_{R_v=0}^{R_{max}} \int_{R_u=0}^{R_v} \int_{d=0}^{R_u} \frac{R_v}{R_{max}} \times \frac{1}{R_{max}} \times \lambda\pi d \times P_{R_u \leq R_v}^{bi}(d, R_u, R_v) d d (d R_u) d R_v \\ &\quad + \int_{R_v=0}^{R_{max}} \int_{R_u=R_v}^{R_{max}} \int_{d=0}^{R_u} \frac{R - R_v}{R_{max}} \times \frac{1}{R_{max}} \times \lambda\pi d \times P_{R_u > R_v}^{bi}(d, R_u, R_v) d d (d R_u) d R_v \quad (8) \end{aligned}$$

a) *Case 1:  $R_u \leq R_v$* : We first note that if  $d \geq R_u$ ,  $P_{R_u \leq R_v}^{bi}(d) = 0$  since  $v$  is not a neighbor of  $u$  at a time  $t_1$ . Next, we find that there exists a value  $d_{min} < R_{max}$  such that, if  $d < d_{min}$ , nodes  $u$  and  $v$  were already neighbors at the time  $t_0$  regardless of  $\alpha$  and  $\beta$  i.e.  $v$  cannot be a new neighbor of  $u$ . Justification and computation details are given in Appendix VIII (Eq. VIII-A). We have that  $d_{min} = \max(0, R_u - 2\Delta d)$  which leads to:

$$P_{bi}(d, R_u, R_v) = \begin{cases} \int_{-\pi}^{\pi} P_{R_u \leq R_v}^{bi}(d, R_u, R_v) d\alpha & \text{if } d_{min} < d < R_u \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where  $P_{R_u \leq R_v}^{bi}(d, R_u, R_v)$  is the probability that node  $v$  at distance  $d$  with radius  $R_v$  is a new bilateral neighbor of node  $u$  with radius  $R_u$  coming from direction  $\alpha$ , assuming  $d_{min} < d < R_u$ .

Now, we can compute  $P_{R_u \leq R_v}^{bi}(d, R_u, R_v)$ . For this purpose, we can notice that, for a given value of  $d_{min} < d < R_u$ , there exists a value  $\alpha_{min}$  such that, for  $\alpha < \alpha_{min}$ , node  $v$  was a bilateral neighbor of node  $u$  at  $t_0$  regardless of its direction  $\beta$ , thus it cannot be a new neighbor. This is illustrated on figure ???. Indeed, for  $\alpha < \alpha_{min}$ , the whole circle  $C_{v_1, \Delta d}$  lies inside the circle  $C_{u_0, R_u}$  thus, for any direction that the node  $v$  may had it was already in the bilateral neighborhood of the node  $u$ . As a result we have:

$$P_{bi}(d, R_u, R_v) = 0 \quad \text{if } \alpha < \alpha_{min} \quad (10)$$

where  $\alpha_{min} = \arccos(\frac{d^2 + 2R_u \Delta d - R_u^2}{2d\Delta d})$ . Computation of  $\alpha_{min}$  is given in Appendix VIII Eq. VIII-B.

For any  $\alpha$ , such that  $\alpha > \alpha_{min}$ , computing the probability  $P_{R_u \leq R_v}^{bi}(d, R_u, R_v)$  is equal to the computing the probability that node  $v$  is coming from the dotted blue angular sector on Fig. ??. Node  $v$  is a new neighbor of node  $u$  if and only if  $\beta$  is such that  $|u_0 v_1| > R_u$  i.e. such that  $v_0$  is outside of the circle  $C_{u_0, R_u}$ . In this case node  $v$  is the new neighbor of  $u$  if and only if  $\beta^- < \beta < \beta^+$ , where  $\beta^-$  and  $\beta^+$  are the angles of the intersection points between  $C_{u_0, R_u}$  and  $C_{v_1, \Delta d}$ , as illustrated on Fig. ??. As a result we have:

$$P_{R_u \leq R_v}^{bi}(d, R_u, R_v) = \int_{\beta^-}^{\beta^+} \frac{d\beta}{2\pi} = \frac{\beta^+ - \beta^-}{2\pi} = \frac{1}{\pi} \arccos(\frac{R_u^2 - \Delta d^2 - k^2}{2d\Delta d})$$

if  $\alpha > \alpha_{min}$  where  $k = |u_0 v_1| = \sqrt{\Delta d^2 + d^2 - 2d\Delta d \cos \alpha}$ . The details of computation of  $\beta^-$ ,  $\beta^+$  and  $k$  are given in Appendix VIII Eq. 23. Thus, when  $d > d_{min}$ :

$$P_{R_u \leq R_v}^{bi}(d, R_u, R_v) = \begin{cases} \frac{1}{\pi} \arccos(\frac{R_u^2 - \Delta d^2 - k^2}{2d\Delta d}) & \text{if } \alpha > \alpha_{min} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

From Eq. 11, we derive:

$$P_{R_u \leq R_v}^{bi}(d, R_u, R_v) = 2 \times \int_0^{\pi} \frac{P_{R_u \leq R_v}^{bi}(d, R_u, R_v, \alpha)}{\pi} d\alpha$$

$$= \begin{cases} \frac{2}{\pi^2} \int_{\alpha_{min}}^{\pi} \arccos(\frac{R_u^2 - \Delta d^2 - k^2}{2d\Delta d}) d\alpha & \text{if } \max(0, R_u - 2\Delta d) \leq d \leq R_u \\ 0 & \text{otherwise} \end{cases}$$

b) *Case 2:  $R_u > R_v$* : Computing of  $P_{R_u > R_v}^{bi}(d, R_u, R_v)$  is similar to the computing of the  $P_{R_u \leq R_v}^{bi}(d, R_u, R_v)$ . The differences are in the values of  $d_{min}$ ,  $\beta^+$ ,  $\beta^-$  and  $\alpha_{min}$  as shown in Appendix.

$$P_{R_u > R_v}^{bi}(d, R_u, R_v) = \begin{cases} \frac{2}{\pi^2} \int_{\alpha_{min}}^{\pi} \arccos(\frac{R_v^2 - \Delta d^2 - k^2}{2k\Delta d}) d\alpha & \text{if } \max(0, R_u - 2\Delta d) \leq d \leq R_u \text{ \& } R_v - 2\Delta d < R_u \\ 0 & \text{otherwise} \end{cases}$$

2) *Computing the number of new unilateral neighbors  $E[N_{uni}]$* : Value that is interesting for our analysis is the probability  $P^{uni}$  that at time  $t_0$ , when  $u$  and  $v$  were not neighbors with given  $R_u$ ,  $R_v$  and  $d$ . We distinguish two cases:

- Case 1:  $R_u \leq R_v$ , noted  $P_{R_u \leq R_v}^{uni}$
- Case 2:  $R_u > R_v$ , noted  $P_{R_u > R_v}^{uni}$

From these two cases we have:

$$E[N_{bi}] = \int_{R_u=0}^R \int_{R_v=0}^R \int_{d=0}^{R_u} P(R_u)P(R_v)\lambda\pi d \times P^{uni}(d, R_u, R_v) d d d R_u d R_v =$$

$$= \int_{R_u=0}^{R_v} \int_{R_v=0}^R \int_{d=0}^{R_u} \frac{R_v}{R} \times \frac{1}{R} \times \lambda\pi d \times P_{R_u > R_v}^{uni}(d, R_u, R_v) d d d R_u d R_v +$$

$$+ \int_{R_u=R_v}^R \int_{R_v=0}^R \int_{d=0}^{R_u} \frac{R - R_v}{R} \times \frac{1}{R} \times \lambda\pi d \times P_{R_u > R_v}^{uni}(d, R_u, R_v) d d d R_u d R_v$$

a) *Case 1:  $R_u \leq R_v$* : In such a case, probability for node  $v$  to be a new unilateral neighbor of node  $u$  is null. Indeed, since  $R_u \leq R_v$ , this is indeed  $u$  which is a bilateral neighbor of node  $v$  and not the opposite, thus we have:

$$P_{R_u > R_v}^{uni}(d, R_u, R_v) = 0 \quad (12)$$

b) *Case 2:  $R_u > R_v$* : We compute the probability  $P_{R_u > R_v}^{uni}$  for the node  $v$  to be a new unilateral neighbor of node  $u$  knowing  $R_u$ ,  $R_v$  and  $d$ . This study is similar to the one given for the bilateral neighbors with the differences in values for  $d_{min}$ ,  $\beta^+$ ,  $\beta^-$  and  $\alpha_{min}$  as shown in Appendix.

$$P_{R_u > R_v}^{uni}(d, R_u, R_v) = \begin{cases} \frac{2}{\pi^2} \int_{\arccos(\frac{d^2 + 2R_u \Delta d - R_u^2}{2d\Delta d})}^{\pi} \arccos(\frac{R_v^2 - \Delta d^2 - k^2}{2k\Delta d}) d\alpha & \text{if } \max(R_v, R_u - 2\Delta d) \leq d \leq R_u \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

### C. Theoretical analysis on minimization of energy cost

In this section, we analyze jointly the optimal frequency and transmission range in order to define the  $f$  function used in Algo. 2.

Hello frequency is also depending of transmission range. The higher transmission range, the lower Hello frequency. In order to find appropriate transmission range we need to find the couple  $f, R$  that gives the minimum of the cost considered in Eq. 5.

$$\frac{\partial \text{cost}(R)}{\partial R} = \frac{\partial}{\partial R} (\Delta t f(R)(R^\alpha + C)) \quad (14)$$

Partial derivative given is applied in order to get the minimum of the cost function:

$$\frac{\partial \text{cost}(R)}{\partial R} = \Delta t \alpha f(R) \times R^{\alpha-1} + \Delta t \frac{\partial f(R)}{\partial R} \times (R^\alpha + C) \quad (15)$$

minimum of this function is obtained when

$$\frac{\partial \text{cost}}{\partial R} = 0 \quad (16)$$

which gives us

$$\frac{\partial f(R)}{\partial R} = -\alpha f(R) \frac{R^{\alpha-1}}{R^\alpha + C} \quad (17)$$

This differential equation gives as a solution:

$$f(R) = \frac{C_1}{R^\alpha + C} \quad (18)$$

where  $C_1$  is a constant defined by initial conditions. For parameters used later on in our simulations, *e.g.*  $\alpha = 4$ ,  $C = 10^8$  given in [13] gives  $C_1 = 2 \times 10^8$ . It becomes:

$$f(R) = \frac{200000000}{R^4 + 100000000} \quad (19)$$

Optimum function  $f(R)$  for energy consumption is then depicted on Figure 2. Note that, as expected, the Hello frequency decreases when the range increases.

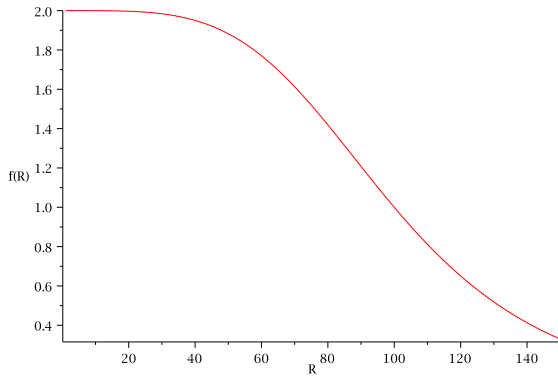


Fig. 2. Solution of differential equation obtained for optimal energy cost.

By replacing  $f$  in Eq. 1, we then need to solve  $R^\alpha - \frac{\alpha R C_1}{2S} + C = 0$  for different  $\alpha$  and  $C$  values.

## VI. SIMULATION RESULTS

We obtained our results using WSNET<sup>2</sup> simulator. We use a model in which the range of transmission can be adapted according to the values that we retrieve using proposed algorithms. In our simulation setup 100 simulation nodes are randomly deployed in a rectangular area of  $1000 \times 1000m$ . Mobility model used for the nodes uniform linear velocity with perfect deflection *i.e.* billiard model of motion.

Important parameters for neighborhood discovery and nodes behavior are followed and their dependence with the respect of the nodes relative speed is given. In the following graphs we present values for the range of transmission, period of HELLO

messages, state of the nodes batteries, observed turnover and accuracy of obtained results are given as the functions of the nodes' speed. In these graphs we refer to the algorithms in the following way: Algorithm 2 is called *NoTAP* since it does not use turnover in calculations, Algorithm 1 with usage of optimal frequency is called *Fopt*, TAP algorithm [5] is referred as TAP while Algorithm 1 with minimized cost is called *Cost*. We will present results retrieved from the simulations with the comment on the performance of the TAP, Fopt and NoTAP algorithm. Separate discussion will be given on the obtained results with the Cost algorithm for adaptation of the range using minimized energy consumption because results obtained using this algorithm are unrealistic (too good) and deserve additional explanation.

Figure 3(a) shows range adaptation as the function of speed of the nodes. We can notice that transmission range for the TAP algorithm is held on the same level, using the same approach (unit disk graph) as in [5]. In the case for Fopt and NoTAP algorithm ranges are increasing as the speed is increasing with the difference that NoTAP algorithm increases range linearly. This behavior of these two algorithms is expected because when nodes are moving faster then they are also changing their neighbors faster so in order to maintain the number of new neighbors the algorithms are increasing the range. We have to note also that Fopt increases the range more than NoTAP since its range is not bounded to single value for a given speed as it is the case for NoTAP. On the other side smaller increase of range of NoTAP algorithm is compensated (as given in Eq.1) with the increase of the period of HELLO messages (Fig.3(b)). Fopt is keeping the period on almost constant value because the adaptation to the higher speed is done with an increase of the range. TAP is decreasing HELLO period with the increase of speed because its range is fixed and the only way to adapt its behavior when nodes are moving faster is to lower the period (hence increase the frequency of HELLO messages). From Figure 3(c) we can see that all algorithms better balance energy than TAP which keeps transmission range on the same and due to this fact has worse results than others. We can also observe that energy loss is bigger as the speed of the nodes increases this is due to the adaptation of algorithms and their attempt to balance the values of turnover, frequency and range which compensates in higher energy loss.

In this case we have to point out that we used linear discharge model for the battery, decreasing certain amount energy from the battery multiplied by range of transmission each time when we transmit packet, constant value for each received packet and loss of energy in idle mode represented by constant value multiplied by the time spent in idle mode (the time between receptions or between reception and transmit and vice versa). This representation of battery is not the best one since it overstates the impact of transmission range which is in real case smaller and is given with the increase of power of transmission.

The results for turnover, shown on Figure 3(d), are more straightforward, we can see that for the algorithms that take

<sup>2</sup><http://wsnet.gforge.inria.fr/>

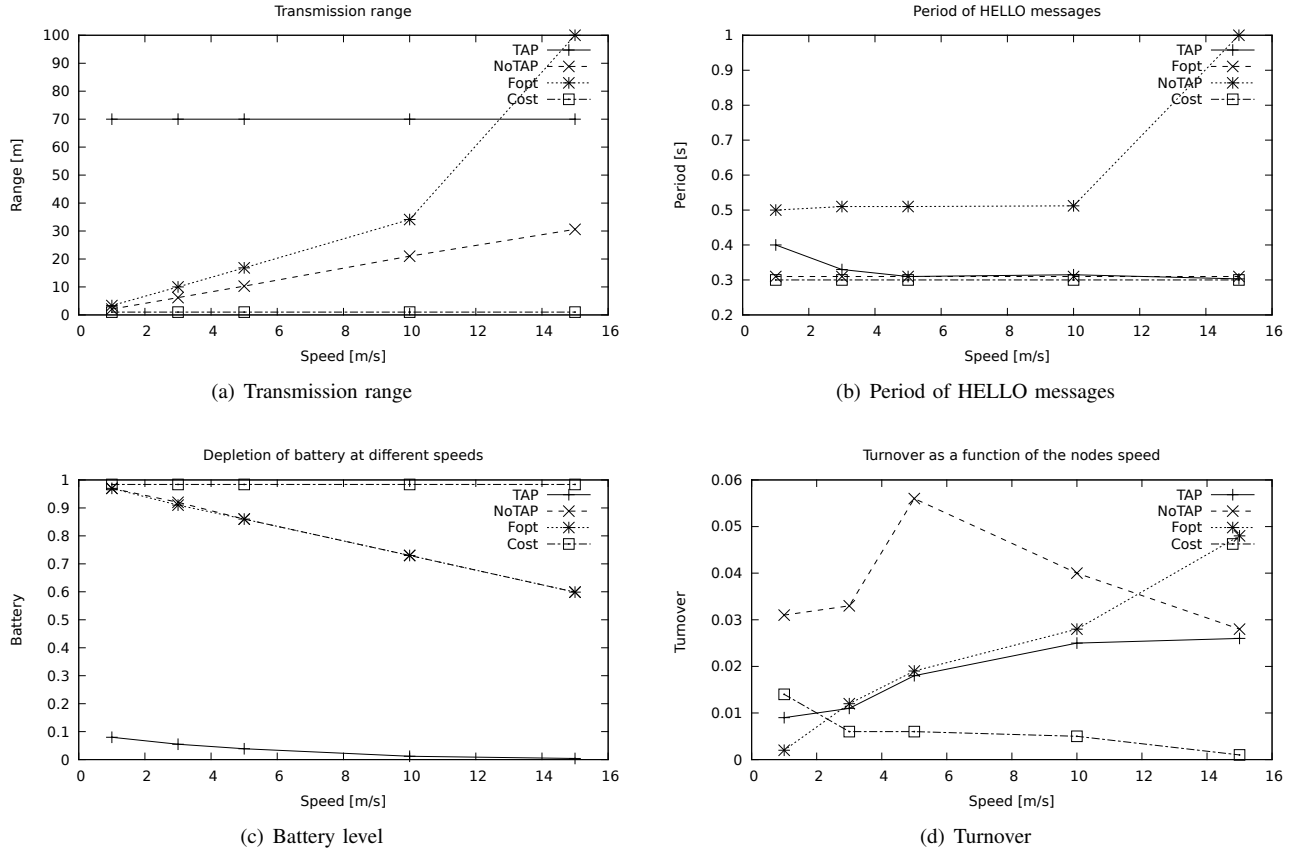


Fig. 3. Different parameters of the WSN as the function of nodes speed

turnover (Fopt and TAP) into account there is an increase of turnover with the increase of speed while NoTAP is having changes of turnover not depending on the speed. Fopt has slightly bigger change of turnover as a result of adaptation both frequency and range.

#### A. Comment on the Cost algorithm

Cost algorithm shows almost perfect results when taking into account state of the battery and accuracy of the neighborhood table, but looking into dependency of the turnover and range gave us the clue what is the reason for this. This algorithm tries to minimize the energy, and its doing it well, but at the cost of keeping the range of transmission on the lowest value thus gaining new neighbors occasionally and with big accuracy. Also since the transmission range is minimal it also preserves battery in the best way. What can we conclude from this is that the model that we imposed for this algorithm is too ideal and does not take into account the nuances this adaptation might have. Minimal transmission range also have another consequence, keeping transmission range on such a low value is ensuring low power consumption but at the same time threatens the correct neighborhood discovery since each node with such a small transmission range detects small number of neighbors and keep its neighborhood table almost empty.

One of the most important values to follow is the accuracy of the neighborhood tables, shown on the Figure 4. Accuracy is calculated checking the state of neighborhood table periodically and checking if all the nodes listed as the neighbors are still in the neighborhood of the node that is being checked. All algorithms are showing tendency of increasing accuracy with increase of the speed which is logical since the faster nodes spend less time in physical neighborhood of their neighboring node.

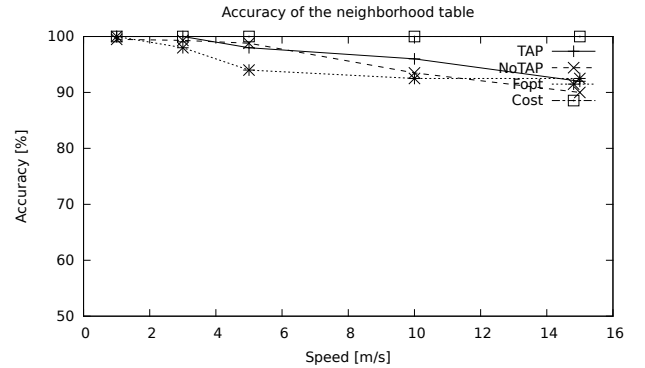


Fig. 4. Accuracy of the neighborhood table as function of speed



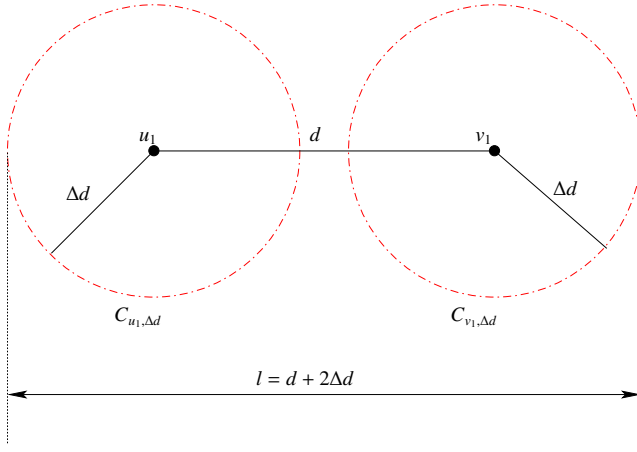


Fig. 5. Calculating  $d_{min}$

## VII. CONCLUSION

In this paper we present neighborhood discovery algorithms that take into account both frequency of hello messages and the transmission range in order to save energy. We are using results obtained by the theoretical analysis on the neighborhood discovery to calculate turnover and its impact on the overall energy consumption. These results are combined with optimal frequency of hello messages for mobile nodes and to conceive two algorithms which are adapting both frequency of hello messages and transmission range. Simulations have proven that our algorithms outperform TAP (which only adapts frequency with fixed range) in terms of energy consumption while keeping the accuracy of the neighborhood table.

Future work would include further improvement of proposed algorithms, taking into account total number of neighbors and received messages thus ensuring both energy efficient and realistic neighborhood discovery (limiting the transmission range to a higher values). Also, since transmission range cannot be directly manipulated on the real sensor hardware connection between transmission range and power of transmission can be made allowing these algorithms to be run on the real wireless sensor networks.

## VIII. APPENDIX

### A. Computing $d_{min}$

We need to compute the mean number of new neighbors node  $u$  detects for a  $\Delta t$  time period. For it, we suppose a node  $v$  at a distance  $d$  from node  $u$  at time  $t_1 = t_0 + \Delta t$ , such that  $0 \leq d \leq R$ .

1) *Case 1 – new bilateral neighbors:*  $v$  is a new bilateral neighbor of node  $u$  which means that  $|u_0 v_0| > \min(R_u, R_v)$  and  $|u_1 v_1| < \min(R_u, R_v)$ .

The value  $d_{min}$  is such that, if  $d < d_{min}$ , nodes  $u$  and  $v$  are neighbors at time  $t_0$  whatever the directions  $\alpha$  and  $\beta$ . As illustrated by the Fig. 5, the longest distance  $l$  between  $u_0$  and  $v_0$  at time  $t_0$  is when both node directions are opposite (for example when  $\alpha = 0$  and  $\beta = \pi$ ). We have  $l = 2\Delta d + d$ , where

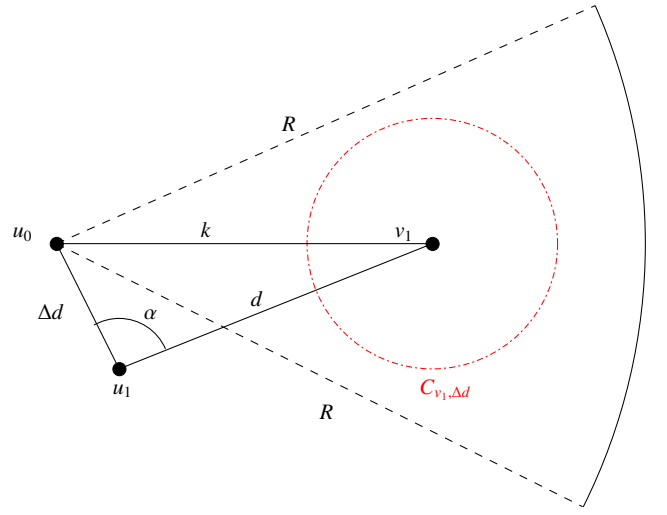


Fig. 6. Calculating  $\alpha_{min}$

$\Delta d = \Delta t \times S$ . For  $v$  to be a new neighbor of node  $u$ , we need  $l > \min(R_u, R_v)$ , which leads to:

$$\begin{aligned} 2\Delta d + d &> \min(R_u, R_v) \\ \Leftrightarrow d &> 2\Delta d - \min(R_u, R_v) \\ \Leftrightarrow d_{min} &= 2\Delta d - \min(R_u, R_v) \end{aligned} \quad (20)$$

Since, depending of the node speed, we may have  $2\Delta d < \min(R_u, R_v)$ , we finally get:

$$d_{min} = \max(0, 2\Delta d - \min(R_u, R_v)) \quad (21)$$

2) *Case 2 – new unilateral neighbors:* By definition, for  $v$  to be a new unilateral neighbor of node  $u$ ,  $d$  is such that  $d > R_v$ . Then, similarly to the previous case,  $d_{min}$  is such that if  $d > d_{min}$ , nodes  $u$  and  $v$  were already neighbors at the time  $t_0$ , whatever the values  $\alpha$  and  $\beta$ .

Since, depending on the node speed, we may have  $2\Delta d < \min(R_u, R_v)$ , we finally get:

$$d_{min} = \max(R_v, 2\Delta d - \min(R_u, R_v)) \quad (22)$$

### B. Computing $\alpha_{min}$

In this section, we are interested in computing the value  $\alpha_{min}$  such that, given  $d > d_{min}$  and  $\alpha$ , if  $\alpha < \alpha_{min}$  then for any  $\beta$  (no matter the value which it takes)  $v$  was neighbor of  $u$  at time  $t_0$ .

For it, we introduce  $k$  such that  $k = |u_0 v_1|$ , like illustrated in Fig 6. According to Pythagore's theorem, we can compute the value of  $k$ :

$$\begin{aligned} \Delta d^2 - (\Delta d \cos(\pi - \alpha))^2 &= k^2 - (d + \Delta d \cos(\pi - \alpha))^2 \\ \Leftrightarrow k^2 &= \Delta d^2 + d^2 - 2d\Delta d \cos \alpha \\ \Leftrightarrow k &= \sqrt{\Delta d^2 + d^2 - 2d\Delta d \cos \alpha} \end{aligned} \quad (23)$$

1) *Case 1 – new bilateral neighbors:* For  $v$  to be new bilateral neighbor of node  $u$  we need that  $|u_0 v_0| > \min(R_u, R_v)$  for any  $\beta$  i.e. that the disks delimited by  $C_{v_1, \Delta d}$  and  $C_{u_0, \min(R_u, R_v)}$  overlap:



